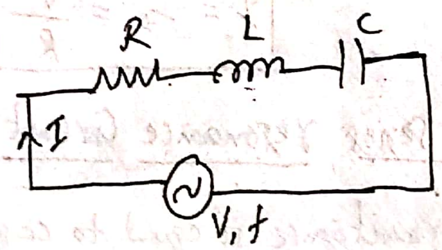
5.1 Definition: An electrical circuit is said to undergo resonance when the net (or total) current is in phase with the applied voltage. Under the condition of resonance the circuit will behave like a pure resistance (the resultant reactance is zero) and the power factor will be unity.

Resonance condition is achieved either by varying frequency and keeping circuit elements constant or by varying circuit elements and keeping the frequency constant.

In a resonant circuit equal amount of energy are interchanged periodically between L & C. The power drawn from the source is only to provide for the energy dissipation in the resistance. Hence a circuit in resonance stores a constant amount of energy.

5.2 Series resonance circuit:



Impedance of the circuit, $Z = R + j(X_L - X_C)$

By varying the supply frequency X_L is made equal to X_C. Then the circuit is said to be in resonance. $Z = R$, the power factor is unity, V & I are in phase. The frequency at which resonance occurs is called as resonant frequency, f_0.

To obtain the resonant frequency equate the imaginary part of the total impedance to zero.

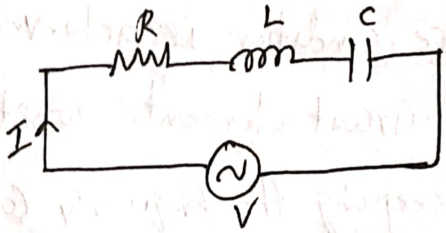
$$X_L - X_C = 0$$

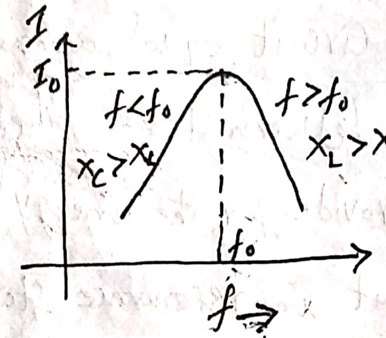
$$X_L = X_C$$

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

5.3 Frequency response of a series RLC Circuit





At resonance current is maximum and is given by

$$\boxed{I = I_0 = \frac{V}{R}}$$

5.4 Properties of series resonance Circuit

1) Inductive reactance is equal to capacitive reactance ($X_L = X_C$)
2) Voltage across inductance is equal to voltage across capacitance ($V_L = V_C$)
3) Current is maximum. ($I_0 = \frac{V}{R}$)
4) Impedance of the circuit is resistive and minimum ($Z = R$)
5) Powerfactor is unity. ($\cos\phi = 1$).

5.5. Voltage magnification (Q factor) (quality factor)

The ratio of the voltage developed across L or C to the applied Voltage is termed as voltage magnification.

$$Q = \frac{V_L}{V} \quad \text{or} \quad Q = \frac{V_C}{V}$$

$$Q = \frac{IX_L}{V} \quad \text{or} \quad Q = \frac{IX_C}{V}$$

At resonance, $V = I_0 R$

$$Q_0 = \frac{I_0 X_L}{I_0 R} \quad \text{or} \quad Q_0 = \frac{I_0 X_C}{I_0 R}$$

$$Q_0 = \frac{X_L}{R} \quad \text{or} \quad Q_0 = \frac{X_C}{R}.$$

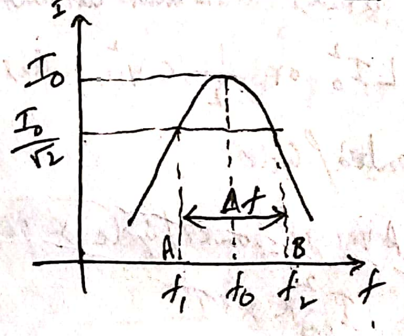
$$\boxed{Q_0 = \frac{\omega_0 L}{R}} \quad \text{or} \quad \boxed{Q_0 = \frac{1}{\omega_0 C R}}$$

we know $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore Q_0 = \frac{L}{\sqrt{LC}\,R} = \frac{\sqrt{L}\sqrt{L}}{\sqrt{L}\sqrt{C}\,R} = \frac{1}{R}\sqrt{L/C}$$

5.6. Bandwidth, selectivity & Q factor



(For definitions, frequency response curve should be written)

Consider the frequency response curve of a series circuit. Let I_0 denote the current at resonance.

The power delivered to the circuit at resonance $= I_0^2 R$ watts

when the current has fallen to $\frac{I_0}{\sqrt{2}}$ i.e; $0.707 I_0$ we have power delivered $= \left(\frac{I_0}{\sqrt{2}}\right)^2 = \frac{1}{2} I_0^2 R$ = Half of the power delivered at resonance

Hence the points A & B on the horizontal frequency axis are termed as half power points, and frequencies f_1 & f_2 at these points are called half power frequencies. f_1 is the lower half power frequency and f_2 is the upper half power frequency.

The range (or band) of frequencies lying between f_1 & f_2 is termed as band width.

$$B.W = \Delta f = (f_2 - f_1)$$

Q factor is defined as the ratio of resonant frequency to bandwidth.

$$Q_0 = \frac{f_0}{f_2 - f_1}$$

Q is defined on energy basis

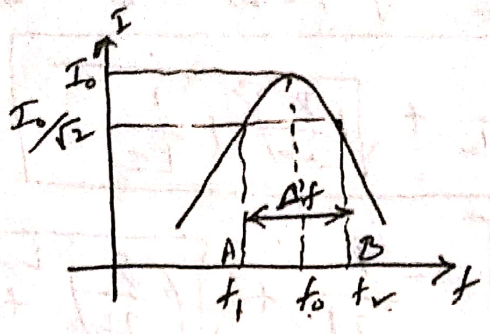
$$Q = 2\pi \left(\frac{\text{max energy stored}}{\text{energy dissipated / cycle}} \right)$$

Selectivity is defined as the ratio of bandwidth to resonant frequency.

$$\text{Selectivity} = \frac{f_2 - f_1}{f_0}$$

$$\text{Selectivity} = \frac{1}{Q_0}$$

5.8. Expression for half power frequencies and Band width



Let f_0 be the resonant frequency and f_1 & f_2 be the lower and upper half power frequencies. At resonance

$$I_0 = \frac{V}{R}$$

At half power frequency $X_L - X_C = R$

At lower half power frequency f_1, $X_C > X_L$

$$X_{C_1} - X_{L_1} = R \quad -(1)$$

At upper half power frequency f_2, $X_L > X_C$

$$X_{L2} - X_{C2} = R \quad -(2)$$

From (1) $X_{C1} - X_{L1} = R$

$$\frac{1}{\omega_1 C} - \omega_1 L = R$$

$$\frac{1 - \omega_1^2 LC}{\omega_1 C} = R$$

$$1 - \omega_1^2 LC = \omega_1 RC$$

$$\omega_1^2 LC + \omega_1 RC - 1 = 0$$

$$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0 \quad \text{(Dividing by } LC)$$

$ax^2 + bx + c = 0$

$ax^2 + bx + c$ → ω_1

$a = 1$

$b = R/L$

$c = -1/LC$

It is of the form $ax^2+bx+c=0$ $a=1,\ b=\frac{R}{L},\ c=-\frac{1}{LC}$

$$\therefore \omega_1 = \frac{-R/L \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4(1)(-1/LC)}}{2\times 1}$$

$$\omega_1 = \frac{-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + 4\cdot\frac{1}{LC}}}{2}$$ (neglecting negative sign as ω_1 cannot be negative)

$$\omega_1 = -\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + 4\cdot\frac{1}{LC}}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\frac{1}{4}\left(\frac{R}{L}\right)^2 + \frac{1}{4}\cdot 4\cdot\frac{1}{LC}}$$

$$\boxed{\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}} \quad -③$$

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$f_1 = -\frac{R}{4\pi L} + \frac{1}{2\pi}\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$f_1 = \frac{-R}{4\pi L} + \sqrt{\frac{1}{4\pi^2}\times\frac{R^2}{4L^2} + \frac{1}{4\pi^2}\cdot\frac{1}{LC}}$$

$$\boxed{f_1 = \frac{-R}{4\pi L} + \sqrt{\left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}}} \quad -④$$

From ② $X_{L2} - X_{C2} = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_2^2 LC - 1 = \omega_2 RC$$

$$\omega_2^2 LC - \omega_2 RC - 1 = 0$$

$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

Solving for ω_2

$$\boxed{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}} \quad -⑤$$

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$\boxed{f_2 = \frac{R}{4\pi L} + \sqrt{\left(\frac{R}{4\pi L}\right)^2 + \frac{1}{4\pi^2 LC}}} \quad -(6)$$

Bandwidth, $\Delta f = f_2 - f_1$ $\quad$ (6 − 4)

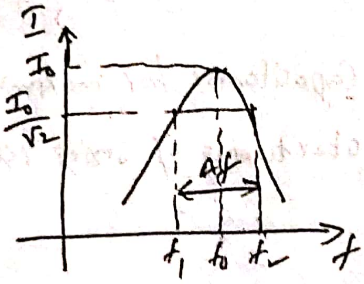
$$\Delta f = \frac{R}{4\pi L} + \frac{R}{4\pi L}$$

$$\Delta f = \frac{2R}{4\pi L}$$

$$\boxed{\Delta f = \frac{R}{2\pi L}}$$

$$Q_0 = \frac{f_0}{f_2 - f_1} = \frac{f_0}{\frac{R}{2\pi L}} = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R}$$

5.9. To prove $f_0 = \sqrt{f_1 f_2}$



Let f_0 be the resonant frequency and f_1 & f_2 be the lower & upper half power frequencies. ω_1 & ω_2 are the angular frequencies.

At lower half power frequency f_1, $X_C > X_L$

$$X_{C_1} - X_{L_1} = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = R \quad -(1)$$

At upper half power frequency f_2, $X_L > X_C$

$$X_{L_2} - X_{C_2} = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad -(2)$$

Subtracting eq 1 - 2

$$\frac{1}{\omega_1 C} - \omega_1 L - \omega_2 L + \frac{1}{\omega_2 C} = 0$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_1 L + \omega_2 L$$

$$\frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = L(\omega_1 + \omega_2)$$

$$\frac{1}{C}\left(\frac{\cancel{\omega_1 + \omega_2}}{\omega_1 \omega_2}\right) = L \cdot (\cancel{\omega_1 + \omega_2})$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

We have $\omega_0 = \frac{1}{\sqrt{LC}}$ at resonance

$$\omega_0^2 = \frac{1}{LC}$$

$$\therefore \omega_1 \omega_2 = \omega_0^2$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\boxed{f_0 = \sqrt{f_1 f_2}}$$

Resonant frequency is the geometric mean of the two half power frequencies.

5.14 List of formulae to be memorised before solving the problems on series resonance

1) $\omega_0 = \dfrac{1}{\sqrt{LC}}$, $\omega_0 = \sqrt{\omega_1 \omega_2}$ $\quad \omega_0, \omega_1, \omega_2$ in rad/sec

2) $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad f_0 = \sqrt{f_1 f_2}$ $\quad f_0, f_1, f_2$ in Hz

3) $I_0 = \dfrac{V}{R}$ (at resonance) $\quad I = \dfrac{V}{Z}$

$(X_L = X_C)$ $(V_L = V_C)$

$(Z = R)$

4) $Q_0 = \dfrac{V_L}{V} = \dfrac{\omega_0 L}{R}$, $Q_0 = \dfrac{V_C}{V} = \dfrac{1}{\omega_0 C R}$, $Q_0 = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$Q_0 = \dfrac{f_0}{f_2 - f_1}$

5) Selectivity $= \dfrac{1}{Q_0} = \dfrac{f_2 - f_1}{f_0}$

6) Bandwidth, $\Delta f = f_2 - f_1 = \dfrac{f_0}{Q_0}$ & $\dfrac{\omega_0}{Q_0}$ $= \dfrac{R}{2\pi L}$ (in Hz)

$= \dfrac{R}{L}$ (in rad/sec)

7) $f_2 = f_0 + \dfrac{\Delta f}{2}$ $\quad f_2 = \dfrac{R}{4\pi L} + \sqrt{\left(\dfrac{R}{4\pi L}\right)^2 + \dfrac{1}{4\pi^2 LC}}$

$f_1 = f_0 - \dfrac{\Delta f}{2}$ $\quad f_1 = \dfrac{-R}{4\pi L} + \sqrt{\left(\dfrac{R}{4\pi L}\right)^2 + \dfrac{1}{4\pi^2 LC}}$

8) $f_L = \dfrac{f_0}{\sqrt{1 - \dfrac{R^2 C}{2L}}}$, $f_C = f_0\sqrt{1 - \dfrac{R^2 C}{2L}}$

9) $V_L = +jIX_L = +jI\omega L = +jI(2\pi f L) = +j\dfrac{V}{Z}(2\pi f L)$

$(V_L)_0 = +jI_0 X_L = +jI_0 \omega_0 L = +jI_0 2\pi f_0 L = +j\dfrac{V}{R}(2\pi f_0 L)$

$(V_L)_{max} = +jIX_L = +jI\omega L = +jI(2\pi f_L L) = +j\dfrac{V}{Z}(2\pi f_L$

$Z = R + j\omega L - j\frac{1}{\omega C} = R + j2\pi f_L L - j\dfrac{1}{2\pi f_L C}$

15

10) $V_c = -jIX_c = -jI.\frac{1}{\omega C} = -jI\frac{1}{2\pi f C} = -j\frac{V}{Z}\frac{1}{2\pi f C}$

$(V_c)_0 = -jI_0X_c = -jI_0\frac{1}{\omega_0 C} = -jI_0\frac{1}{2\pi f_0 C} = -j\frac{V}{R}\frac{1}{2\pi f_0 C}$

$(V_c)_{max} = -jIX_c = -jI.\frac{1}{\omega C} = -jI.\frac{1}{2\pi f_c C} = -j.\frac{V}{Z}\frac{1}{2\pi f_c C}$

$\hookrightarrow Z = R + j2\pi f_c L - j\frac{1}{2\pi f_c C}$
$(f = f_c)$

Problems on Series resonance

5.1. A Coil of 5mH inductance and 10Ω resistance is Connected in series with 5μF Capacitor. Determine frequency at which Circuit resonates.

Sol: Given: $R = 10\Omega$, $L = 5mH = 5\times10^{-3}H$, $C = 5\mu F = 5\times10^{-6}F$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5\times10^{-3}\times5\times10^{-6}}} = \underline{\underline{1.0065\ kHz}}$$

5.2. In a series RLC Circuit driven with a sinusoidal a.c. voltage source, determine value of C required to achieve resonance in Circuit at 5kHz if value of resistance and inductance are 2Ω and 1mH respectively.

Sol: Given: $R = 2\Omega$, $L = 1mH = 1\times10^{-3}H$, $f_0 = 5\ kHz = 5\times10^{3}Hz$

(Here 'C' is Varied to achieve resonance)

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\sqrt{LC} = \frac{1}{2\pi f_0}$$

$$LC = \frac{1}{4\pi^2 f_0^2}$$

$$C = \frac{1}{4L\pi^2 f_0^2} = \frac{1}{4\times1\times10^{-3}\times\pi^2\times(5\times10^3)^2}$$

$$= \underline{\underline{1.0132\mu F}}$$

5.3. A Coil of inductance 0.1 H and resistance of 10Ω is connected in series with a Capacitor of 0.1 μF. Find frequency of resonance of the Circuit. Also find quality factor of the Circuit at resonance.

Sol: Given: $L = 0.1\,H$, $R = 10\,\Omega$, $C = 0.1\,\mu F$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 0.1 \times 10^{-6}}} = 1.5915\,KHz$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 1.5915 \times 10^3 \times 0.1}{10} \simeq 100$$

5.4. A Circuit Consisting of a resistance of 4Ω, and inductor of 0.5 H and a Variable capacitor all in series connected across 100V, 50 Hz supply. At resonance calculate:

i) Capacitance ii) Voltage across inductor iii) Q-factor iv) Current in the Circuit v) Find current at 100 Hz

Sol: Given: $V = 100V$, $f_0 = 50Hz$, $R = 4\Omega$, $L = 0.5H$

i) $$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 L f_0^2} = \frac{1}{4\pi^2 \times 0.5 \times 50^2}$$

$$= 20.264\,\mu F$$

ii) $$(V_L)_0 = +jI_0X_L$$

$$= +j \cdot \frac{V}{R}\omega_0 L = +j\frac{V}{R}2\pi f_0 L$$

$$= +j\frac{100}{4} \times 2\pi \times 20.2642 \times 10^{-6} \times 0.5 = j3.927\,kV$$

iii) $$Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi \times 50 \times 0.5}{4} = 39.27$$

iv) $$I_0 = \frac{V}{R} = \frac{100}{4} = 25A$$

v) $\underset{(\text{at } 100Hz)}{I} = \frac{V}{Z} = \frac{100}{R+j\omega L - j\cdot\frac{1}{\omega C}} = \frac{100}{R+j2\pi f L - j\frac{1}{2\pi f C}}$

$$= \frac{100}{4+j2\pi\times100\times0.5 - j\cdot\frac{1}{2\pi\times100\times20.2642\times10^{-6}}} = \frac{100}{4+j235.61} = 0.4243\angle{-89.02}\,A$$

5.4. A Series RLC Circuit Consists of $R=100\Omega$, $L=0.02\,H$ and $C=0.02\,\mu F$. Calculate frequency of resonance. A variable frequency sinusoidal voltage of value 50V is applied to the circuit. Find the frequency at which voltage across L & C is maximum. Also calculate voltage across L & C at frequency of resonance. Find maximum current in the circuit.

Sol: Given: $R=100\Omega$, $L=0.02H$, $C=0.02\,\mu F$, $V=50V$

i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times0.02\times10^{-6}}} = 7.957\,KHZ$

ii) $f_C = f_0\sqrt{1-\frac{R^2C}{2L}} = 7.957\times10^3\sqrt{1-\frac{100^2\times0.02\times10^{-6}}{2\times0.02}}$

$= 7.937\,KHZ$

iii) $f_L = \frac{f_0}{\sqrt{1-\frac{R^2C}{2L}}} = \frac{7.957\times10^3}{\sqrt{1-\frac{100^2\times0.02\times10^{-6}}{2\times0.02}}} = 7.977\,KHZ$

iv) $(V_L)_0 = +jI_0X_L = +j\cdot\frac{V}{R}2\pi f_0 L = +j\cdot\frac{50}{100}\times2\pi\times7.957\times10^3\times0.02$

$= j500 = 500\angle 90\,V$

v) $(V_C)_0 = -jI_0X_C = -jI_0\frac{1}{2\pi f_0 C} = -j\frac{50}{100}\times\frac{1}{2\pi\times7.957\times10^3\times0.02\times10^{-6}}$

$= -j500 = 500\angle{-90}\,V$

vi) $I_0 = \frac{V}{R} = \frac{50}{100} = 0.5A$

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5.5. A series RLC circuit consists of a resistance of 1kΩ and an inductance of 100mH in series with capacitance of 10pF. If 100V is applied as input across the combination determine

i) The resonant frequency ii) Maximum current in the circuit

iii) Q-factor of the circuit iv) the half-power frequencies

Sol:

i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times10\times10^{-12}}} = 159.15\,KHz$

ii) $I_0 = \frac{V}{R} = \frac{100}{1\times10^3} = 0.1\,A$

iii) $Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi\times159.15\times10^3\times100\times10^{-3}}{1\times10^3}$

$= 100$

iv) Bandwidth, $\Delta f = \frac{R}{2\pi L} = \frac{1000}{2\pi\times100\times10^{-3}} = 1591.5\,Hz$

v) $f_1 = f_0 - \frac{\Delta f}{2} = 159.15\times10^3 - \frac{1591.5}{2} = 158.35\,KHz$

$f_2 = f_0 + \frac{\Delta f}{2} = 159.15\times10^3 + \frac{1591.5}{2} = 159.95\,KHz$

5.6. A series resonant circuit includes 1μF capacitor and a resistance of 16Ω! If the bandwidth is a 500 rad/sec, determine

i) ω_0 ii) Q iii) L

Sol: Given: $C = 1\mu F$, $R = 16\Omega$, $B.W = 500\,rad/sec$

i) Since B.W is given in rad/sec

$B.W = \frac{R}{L}$

$L = \frac{R}{B.W} = \frac{16}{500} = 32\,mH$

ii) $Q_0 = \frac{\omega_0 L}{R}$ or $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{16}\sqrt{\frac{32\times10^{-3}}{1\times10^{-6}}}$

$= 11.1803$

iii) $B.W = \frac{\omega_0}{Q_0}$

$$\omega_0 = (B.W) Q_0 = 500 \times 11.1803 = 5590.15 \text{ rad/sec}$$

5.6 A series RLC circuit has $R = 50\Omega$, $L = 0.01H$ and $C = 0.04\mu F$ and is connected to a.c. source of 100V. Find the i) Resonant frequency ii) Circuit impedance at resonant frequency iii) maximum value of voltage across capacitance and the frequency at which it occurs iv) Voltage across inductance at resonance.

Sol: $R = 50\Omega$, $L = 0.01H$, $C = 0.04\mu F$, $V = 100V$

i) $$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 0.04 \times 10^{-6}}} = 7.9577 \text{ KHz}$$

ii) $Z_0 = R = 50\Omega$

iii) $$Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 7.9577 \times 10^3 \times 0.01}{50} \simeq 10$$

iv) The frequency at which voltage across C is maximum is given by $f_c = f_0\sqrt{1 - \frac{R^2 C}{2L}}$

$$= 7.9577 \times 10^3 \sqrt{1 - \frac{50^2 \times 0.04 \times 10^{-6}}{2 \times 0.01}}$$

$$= 7.937 \text{ KHz}$$

$$(V_c)_{max} = -jIX_c$$

$$= -jI \cdot \frac{1}{2\pi f_c C} = -j \cdot \frac{V}{Z} \cdot \frac{1}{2\pi f_c C}$$

$$= -j \cdot \frac{V}{R + j2\pi f_c L - j\frac{1}{2\pi f_c C}} \times \frac{1}{2\pi f_c C}$$

$$= -j \cdot \left[\frac{100}{50 + j2\pi \times 7.937 \times 10^3 \times 0.01 - j\frac{1}{2\pi \times 7.937 \times 10^3 \times 0.04 \times 10^{-6}}}\right] \times \frac{1}{2\pi \times 7.937 \times 10^3}$$

$$= 1001.26 \angle -87 \text{ V}$$

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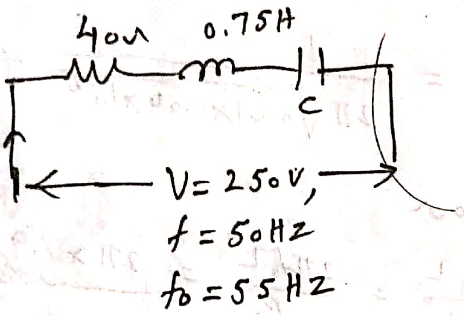
v) $(V_L)_0 = +j I_0 X_L = +j \cdot \frac{V}{R} \cdot \omega_0 L = +j \frac{V}{R} \cdot 2\pi f_0 L$

$= +j \frac{100}{50} \times 2\pi \times 7.9577 \times 10^{3} \times 0.01$

$= 1000 \angle 90^\circ$ V.

5.7. Determine i) the line current ii) the p.f iii) voltage across the coil, when a coil of resistance 40Ω and inductance of 0.75H forms a part of a series circuit, for which the resonant frequency is 55Hz, if the supply is 250V, 50Hz.

Sol:



(In the problem C is not given. It is understood that C has to be present to achieve resonance)

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 55)^2 \times 0.75} = 11.165\,\mu F$

i) $\underset{\text{(Line Current)}}{I} = \frac{V}{Z}$

$Z = R + j2\pi f L - j\frac{1}{2\pi f C}$

$= 40 + j2\pi \times \overset{\text{not } f_0}{50} \times 0.75 - j\frac{1}{2\pi \times 50 \times 11.165 \times 10^{-6}}$

$= (40 - j49.4767)\,\Omega = \underset{|Z|}{63.6325} \angle -51.05$

$\therefore I = \frac{V}{Z} = \frac{250}{(40 - j49.4767)} = 3.93 \angle 51.05$ A

ii) $\cos\phi = \frac{R}{|Z|} = \frac{40}{63.6325} = 0.6286$ (leading)

(Since I is leading w.r.t V)

iii) $V_{Coil} = I\, \underset{(R + j2\pi f L)}{Z_{Coil}} = 3.93 \angle 51.05 \times (40 + j2\pi \times 50 \times 0.75)$

$= 939.23 \angle 131.4$

5.8. It is required that a series RLC circuit should resonate at 1 MHz. Determine values of R, L and C if bandwidth of the circuit is 5 kHz and its impedance is 50 Ω at resonance.

Sol: Given: $B.W = 5\,kHz = 5000\,Hz$

$Z_0 = R = 50\,\Omega$

$f_0 = 1\,MHz = 1\times10^6\,Hz$

i) $R = Z_0 = 50\,\Omega$

ii) $B.W = \frac{R}{2\pi L}$ (Since B.W is given in Hz)

$$L = \frac{R}{2\pi(B.W)} = \frac{50}{2\pi\times5000} = 1.5915\,mH$$

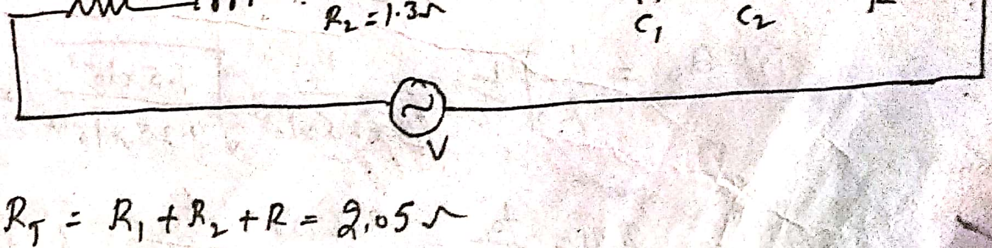
iii) $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$C = \frac{1}{(2\pi f_0)^2(L)} = \frac{1}{(2\pi\times1\times10^6)^2(1.5915\times10^{-3})} = 15.9159\,pF$$

5.9. Two coils; one of $R_1 = 0.51\,\Omega$, $L_1 = 32\,mH$ and other coil of $R_2 = 1.3\,\Omega$, $L_2 = 15\,mH$ are in series and are in series with a capacitor of 25 μF and 62 μF and a series resistor of resistance 0.24 Ω. Determine the following:

i) Resonant frequency ii) Q-factor of the circuit iii) Bandwidth
iv) power dissipated in the circuit at resonant frequency.

Sol:



$R_T = R_1 + R_2 + R = 2.05\,\Omega$

$L_T = L_1 + L_2 = 47\,mH$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_T = \frac{C_1C_2}{C_1+C_2} = 17.816\,\mu F$$

i) $f_0 = \dfrac{1}{2\pi\sqrt{L_T C_T}} = \dfrac{1}{2\pi\sqrt{47\times10^{-3}\times17.816\times10^{-6}}}$

$= 173.9266\ Hz$

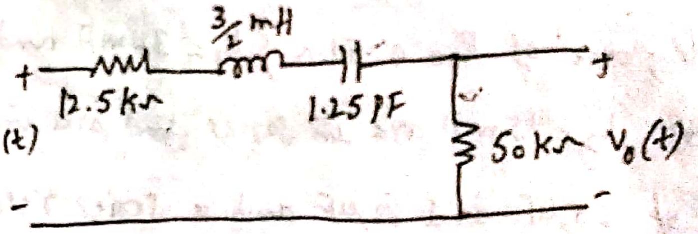
ii) $Q_0 = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{2.05}\sqrt{\dfrac{47\times10^{-3}}{17.816\times10^{-6}}} = 25.0547$

iii) $B.W = \dfrac{R}{2\pi L} = \dfrac{2.05}{2\pi\times47\times10^{-3}} = 6.9418\ Hz$

iv) $I_0 = \dfrac{V}{R} = \dfrac{1}{2.05} = 0.4878A$ ('V' is not given. Assumed as 1V)

$P_0 = I_0^2 R = (0.4878)^2(2.05) = 0.4877W$

5.10 For the network shown in the figure, determine the following
i) f_0 ii) Q iii) Half power frequencies iv) Bandwidth.



Sol: $R_T = 12.5 + 50 = 62.5\ k\Omega$

i) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{1.25\times10^{-3}\times1.25\times10^{-12}}}$

$= 3.6755\ MHz$

ii) $Q_0 = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{62.5\times10^{3}}\sqrt{\dfrac{1.5\times10^{-3}}{1.25\times10^{-12}}} = 0.5542$

iii) $B.W = \dfrac{f_0}{Q_0} = \dfrac{3.6755\times10^{6}}{0.5542} = 6.632\ MHz$

$f_2 = f_0 + \dfrac{\Delta f}{2} = 6.9915\ MHz$

$f_1 = f_0 - \dfrac{\Delta f}{2} = 3.595\ MHz$

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5.11) A Coil is Connected in series with a variable capacitor across $v(t) = 10 \cos 1000t$. The Capacitor is varied and the current is maximum when $C = 10\mu F$. When $C = 12.5\mu F$, the current is 0.707 times the maximum value. Find L, R and Q of the Coil.

Sol:

$$v(t) = 10 \cos 1000t \qquad \therefore \omega_0 = 1000 \text{ rad/sec}$$
$$(10 \cos \omega_0 t) \qquad C = 10\mu F$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$1000 = \frac{1}{\sqrt{L \times 10 \times 10^{-6}}}$$

$$L = 0.1 H$$

At $C = 12.5\mu F$, Current decreases to 0.707 times $(I_0/\sqrt{2})$ max. Current (or power reduces to half). This is half power Condition.

At half power Condition, we can write,

$$|X_L - X_C| = R$$

$$\left|\omega_0 L - \frac{1}{\omega_0 C}\right| = R$$

$$\left|(1000 \times 0.1) - \frac{1}{(1000 \times 12.5 \times 10^{-6})}\right| = R$$

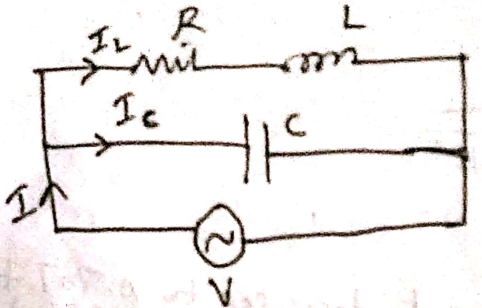
$$R = 20\Omega$$

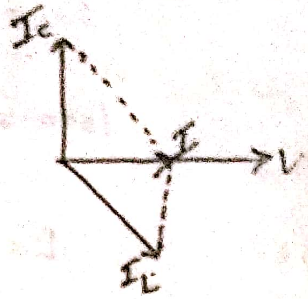
$$Q_0 = \frac{\omega_0 L}{R} = \frac{1000 \times 0.1}{20} = 5$$

5.15 <u>Parallel resonance (Anti resonance)</u>

A parallel circuit is said to be in resonance when applied voltage and resulting current are in phase that gives unity power factor condition.

a) <u>Practical parallel resonant circuit</u>





The impedance of the coil is given by

$$Z_L = R + j\omega L$$

The admittance of the coil is given by

$$Y_L = \frac{1}{Z_L} = \frac{1}{R+j\omega L} \times \frac{R-j\omega L}{R-j\omega L} = \frac{R-j\omega L}{R^2+\omega^2L^2}$$

$$Z_C = -jX_C = -j \cdot \frac{1}{\omega C}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{-j \cdot \frac{1}{\omega C}} = \frac{\omega C}{-j} = +j\omega C$$

The total admittance of the circuit is

$$Y = Y_L + Y_C = \frac{R-j\omega L}{R^2+\omega^2L^2} + j\omega C = \frac{R}{R^2+\omega^2L^2} + j\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)$$

For the circuit to be at resonance, the impedance of the circuit should be purely resistive or the admittance must be purely conductive. Hence, the imaginary part of the admittance must be zero.

$$\therefore \quad \omega_0 C - \frac{\omega_0 L}{R^2+\omega_0^2L^2} = 0$$

$$\omega_0 C = \frac{\omega_0 L}{R^2+\omega_0^2 L^2}$$

$$\boxed{R^2+\omega_0^2 L^2 = \frac{L}{C}} \quad -(1)$$

$$\omega_0^2 = \frac{\frac{L}{C}-R^2}{L^2} = \frac{\frac{L}{C}}{L^2} - \frac{R^2}{L^2}$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\boxed{\omega_0 = \sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}}$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}\left(1-\frac{R^2C}{L}\right)$$

At resonance, the admittance of the circuit is purely conductive

$$Y_0 = \frac{R}{R^2+\omega_0^2L^2} \quad \text{but} \quad R^2+\omega_0^2L^2 = \frac{L}{C} \ (\text{from } 1)$$

$$Y_0 = \frac{R}{\frac{L}{C}} = \frac{RC}{L}$$

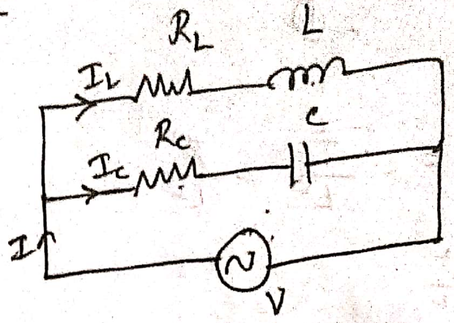
$$Z_0 = \frac{1}{Y_0} = \frac{L}{RC}$$

Z_0 is the impedance of the practical parallel circuit at resonance and is known as the dynamic resistance.

The current is given by $I_0 = VY_0 = V\times\frac{RC}{L}$

$$\boxed{I_0 = \frac{VCR}{L}}$$

b) Parallel Resonant Circuit considering the capacitance to have resistance



The total admittance Y is given by

$$Y = Y_L + Y_c = \frac{1}{R_L + j\omega L} + \frac{1}{R_c - j\cdot\frac{1}{\omega C}}$$

$$= \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R_L - j\omega L} + \frac{1}{R_c - j\cdot\frac{1}{\omega C}} \times \frac{R_c + j\cdot\frac{1}{\omega C}}{R_c + j\cdot\frac{1}{\omega C}}$$

$$Y = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_c + j\cdot\frac{1}{\omega C}}{R_c^2 + \frac{1}{\omega^2 C^2}}$$

$$Y = \left[\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_c}{R_c^2 + \frac{1}{\omega^2 C^2}}\right] + j\left[\frac{\frac{1}{\omega C}}{R_c^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2}\right]$$

At resonance, the admittance is purely conductive. Hence, the imaginary part of 'Y' is zero.

$$\frac{\frac{1}{\omega_0 C}}{R_c^2 + \frac{1}{\omega_0^2 C^2}} = \frac{\omega_0 L}{R_L^2 + \omega_0^2 L^2}$$

$$\frac{1}{\omega_0 C}\left(R_L^2 + \omega_0^2 L^2\right) = \omega_0 L\left(R_c^2 + \frac{1}{\omega_0^2 C^2}\right)$$

$$\frac{1}{LC}\left(R_L^2 + \omega_0^2 L^2\right) = \omega_0^2\left(R_c^2 + \frac{1}{\omega_0^2 C^2}\right)$$

$$\frac{R_L^2}{LC} + \omega_0^2 \frac{L}{C} = \omega_0^2 R_c^2 + \frac{1}{C^2}$$

$$\omega_0^2\left(R_C^2-\frac{L}{C}\right)=\frac{R_L^2}{LC}-\frac{1}{C^2}=\frac{1}{LC}\left(R_L^2-\frac{L}{C}\right)$$

$$\boxed{\omega_0=\frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}} \quad \text{or} \quad \boxed{f_0=\frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}}$$

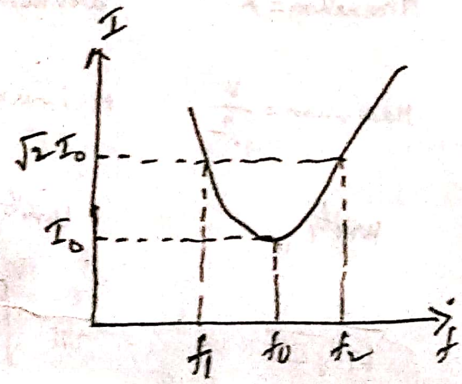
$\boxed{\text{The circuit resonates at all frequencies if } R_L^2=R_C^2=\frac{L}{C}}$

The admittance at resonance is purely conductive

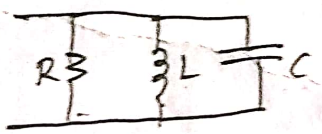
$$Y_0=\frac{R_L}{R_L^2+\omega_0^2L^2}+\frac{R_C}{R_C^2+\frac{1}{\omega_0^2C^2}}$$

$$I_0=VY_0=V\left[\frac{R_L}{R_L^2+\omega_0^2L^2}+\frac{R_C}{R_C^2+\frac{1}{\omega_0^2C^2}}\right]$$

The frequency response curve of a parallel resonant circuit is shown in the figure below.



c) <u>A general parallel resonant circuit</u>



$$Y=Y_1+Y_2+Y_3$$

$$=\frac{1}{R}+\frac{1}{jX_L}+\frac{1}{-jX_C}$$

$$=G-j\frac{1}{X_L}+j\cdot\frac{1}{X_C}$$

$$=G+j\left(\frac{1}{X_C}-\frac{1}{X_L}\right)$$

The conductance of R is $G=\frac{1}{R}$

The susceptance of L is $-jB_L=-j\cdot\frac{1}{X_L}=-j\cdot\frac{1}{\omega L}$

The susceptance of C is $jB_C=j\cdot\frac{1}{X_C}=j\omega C$

The total admittance of the circuit is given by

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

For the circuit to be at resonance, Y should be a pure conductance, Hence, the imaginary part of Y is zero

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

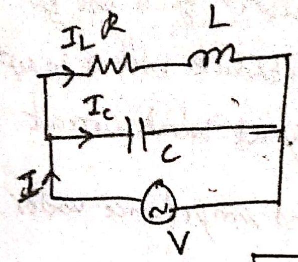
$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

5.16 Comparison between series & parallel resonance

Parameter	Series Circuit	Parallel Circuit
1) Impedance at resonance (Z_0)	Minimum $= R$	Maximum $= \frac{L}{CR}$
2) Current at resonance (I_0)	Maximum $= \frac{V}{R}$	Minimum $= \frac{VCR}{L}$
3) p.f at resonance	unity	unity
4) Resonant frequency, f_0	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
5) Quality factor, Q_0	$\frac{\omega_0 L}{R}$	$\frac{\omega_0 L}{R}$

5.17 List of formulae to be memorised before solving the problems on parallel resonance

a)



$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \; ; \; \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

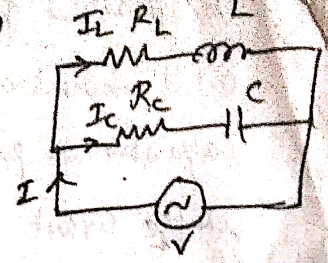
$$Z_0 = \frac{L}{RC} \text{ (dynamic resistance)}$$

$$I_0 = \frac{V}{Z_0} = \frac{VCR}{L} \; ; \; I_L = I_c = Q_0 I_0$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$$B.W = \frac{f_0}{Q_0} = \frac{1}{RC}$$

b)

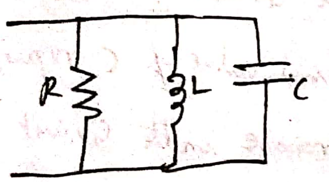


$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_c^2 - \frac{L}{C}}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_c^2 - \frac{L}{C}}}$$

The circuit resonates at all frequencies if $R_L^2 = R_c^2 = \frac{L}{C}$

c)



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\boxed{Q_0 = \frac{R}{\omega_0 L} = \omega_0 CR} \; \boxed{Q_0 = R\sqrt{\frac{C}{L}}}$$

$$B.W = \frac{f_0}{Q_0}$$

Problems on parallel resonance

2. A practical parallel resonant circuit consists of a coil of 0.1 H inductance with 10Ω leakage resistance with a 10μF capacitor in parallel with it. Find frequency at which current in ~~the~~ circuit is purely resistive. Also find impedance under resonance.

Given: $L = 0.1H$, $R_L = 10\Omega$, $C = 10\mu F$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 10 \times 10^{-6}} - \frac{10^2}{(0.1)^2}}$$

$$= 158.35\ Hz$$

$$Z_0 = \frac{L}{RC} = \frac{0.1}{10 \times 10^{-6} \times 10} = 1k\Omega$$

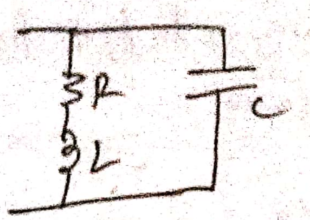
A practical parallel resonant circuit is driven by ac mains supply 230V, 50Hz. Find value of C required to be varied to achieve antiresonance in the circuit if it is shunted with a coil of 1mH inductance and 10Ω resistance.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{1 \times 10^{-3} C} - \frac{10^2}{(1 \times 10^{-3})^2}}$$

$C = 9.99 \mu F$ $\qquad C = \dfrac{1}{\left(4\pi^2 f_0^2 + \frac{R^2}{L}\right)}$

In the circuit given below in figure, an inductance of 0.1H having a Q of 5 is in parallel with a capacitor. Determine the value of capacitance and coil resistance at resonant frequency of 500 rad/sec



Sol:
$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

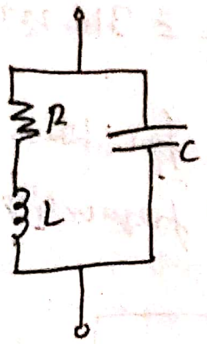
$$Q_0 = \frac{\omega_0 L}{R} = \frac{500 \times 0.1}{R} = 5$$

$$R = 10\,\Omega$$

$$500 = \sqrt{\frac{1}{0.1\,C} - \frac{10^2}{0.1^2}}$$

$$C = 38.4\,\mu F$$

5.14. If $R = 25\,\Omega$, $L = 0.5\,H$, $C = 5\,\mu F$, find ω_0, Q and bandwidth for the circuit as shown in the figure.



Sol:
$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{0.5 \times 5 \times 10^{-6}} - \frac{25^2}{0.5^2}}$$

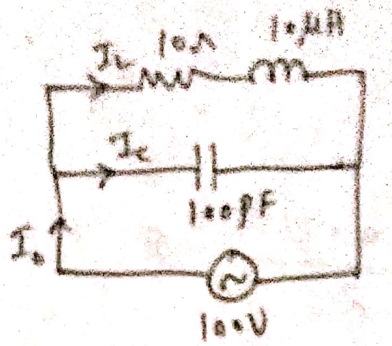
$$f_0 = 100.343\ Hz \quad ; \quad \omega_0 = 630.476\ rad/sec$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{(100.343)\,2\pi \times 0.5}{25} = 12.6095$$

$$B.W = \frac{f_0}{Q_0} = 7.9577\ Hz$$

$$= \frac{\omega_0}{Q_0} = 50\ rad/sec$$

5.15 For a parallel resonant circuit shown in the figure find I_0, I_L, I_C, f_0 and dynamic resistance.



Sol:

i) $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = 5.0304$ MHz

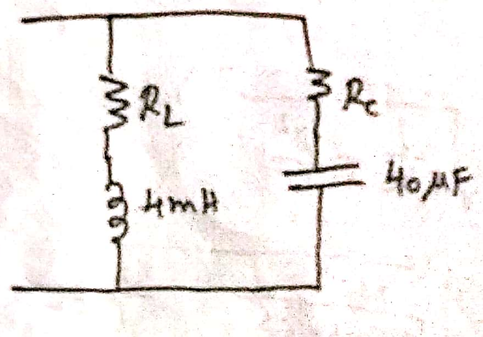
ii) $Z_0 = \frac{L}{RC} = 10\ k\Omega$

iii) $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = 31.6227$

iv) $I_0 = \frac{V}{Z_0} = \frac{100}{10\times10^3} = 10$ mA

v) $I_C = I_L = Q_0 I_0 = 316.227$ mA

5.16 Determine R_L & R_C for which the circuit shown in figure resonates at all frequencies.



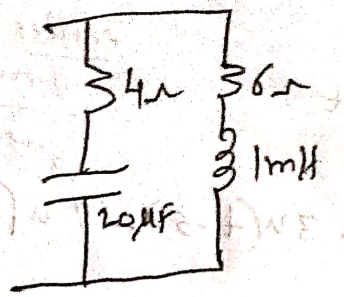
Sol:

To have resonance at all frequencies

$$R_L = R_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{4\times10^{-3}}{40\times10^{-6}}} = 10\Omega$$

(37)

Find the resonant frequency for the two branch parallel ckt in the figure below.



$R_L = 6\Omega$ → resistance in series with L

$R_C = 4\Omega$ → resistance in series with C

Sol:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

$$= \frac{1}{2\pi\sqrt{(1\times10^{-3}\times20\times10^{-6})}}\sqrt{\frac{6^2 - \frac{1\times10^{-3}}{(20\times10^{-6})}}{4^2 - \frac{1\times10^{-3}}{(20\times10^{-6})}}}$$

$$= \frac{1}{2\pi\times1.414\times10^{-4}}\sqrt{\frac{36-50}{16-50}}$$

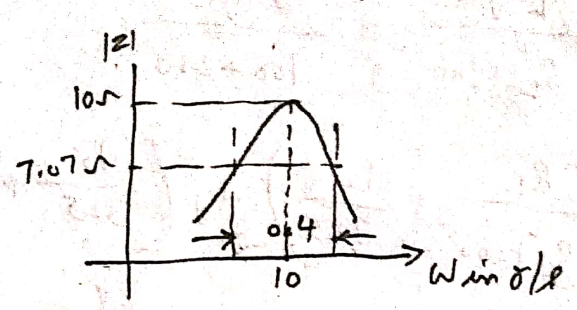
$$= \frac{1}{8.88\times10^{-4}}\sqrt{\frac{-14}{-34}}$$

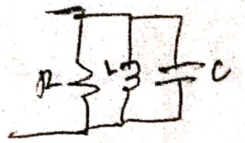
$$= \frac{1}{8.88\times10^{-4}}\sqrt{0.411}$$

$$= \frac{1}{8.88\times10^{-4}}\times0.641$$

$f_0 = 721.84\ Hz$

5.17 Determine the R-L-C parallel circuit parameters whose response curve is as shown in the figure. What are the new values of ω_0 and bandwidth if C is increased 4 times?





$f_0 = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$

Sol: Given: $\omega_0 = 10$ rad/sec

$\Delta f = 0.4$

$R = 10\Omega$

$$Q_0 = \frac{\omega_0}{B.W} = \frac{10}{0.4} = 25$$

$$Q_0 = \frac{R}{\omega_0 L} \Rightarrow L = \frac{R}{\omega_0 Q_0} = \frac{10}{10 \times 25} = 0.04\,H$$

Note not $\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 CR}$

$$Q_0 = \omega_0 CR \Rightarrow C = \frac{Q_0}{\omega_0 R} = \frac{25}{10 \times 10} = 0.25\,F$$

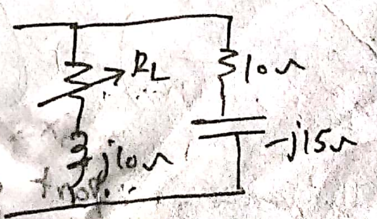
If C is increased by 4 times $\Rightarrow C = 4 \times 0.25 = 1F$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.04 \times 1}} = 5 \text{ rad/sec}$$

$$Q_0 = \omega_0 CR = 5 \times 1 \times 10 = 50$$

$$B.W = \frac{\omega_0}{Q_0} = \frac{5}{50} = 0.1 \text{ rad/sec.}$$

5.18 Find the value of R_L for which the circuit shown in figure is resonant



Sol:

$$Y = \frac{1}{R_L + j10} + \frac{1}{10 - j15}$$

$$Y = \frac{R_L - j10}{R_L^2 + 100} + \frac{10 + j15}{100 + 225}$$

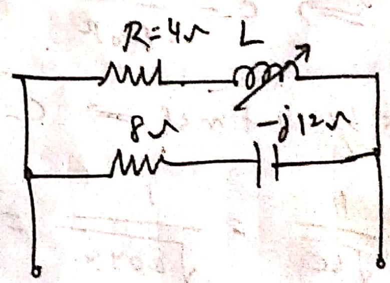
$$= \left(\frac{R_L}{R_L^2 + 100} + \frac{10}{325}\right) + j\left(\frac{15}{325} - \frac{10}{R_L^2 + 100}\right)$$

For the circuit to be at resonance, the imaginary part of Y is zero

$$\frac{15}{325} - \frac{10}{R_L^2 + 100} = 0$$

Solving, we get $R_L = 10.8\,\Omega$

5.19 Find the values of L for which the circuit given in figure resonates at $\omega = 5000$ rad/sec.



Sol:

$$Y = \frac{1}{4 + jX_L} + \frac{1}{8 - j12}$$

$$= \frac{4 - jX_L}{4^2 + X_L^2} + \frac{8 + j12}{8^2 + 12^2}$$

$$= \left(\frac{4}{4^2 + X_L^2} + \frac{8}{8^2 + 12^2}\right) + j\left(\frac{12}{8^2 + 12^2} - \frac{X_L}{4^2 + X_L^2}\right)$$

At resonance, the imaginary part of Y is zero

$$\frac{12}{8^2+12^2} = \frac{X_L}{4^2+X_L^2} \qquad 192+12X_L^2 = 208X_L$$

$$12X_L^2 - 208X_L + 192 = 0$$

Simplifying, we get $3X_L^2 - 52X_L + 48 = 0$

$$X_L = \frac{52 \pm \sqrt{52^2 - 4\times3\times48}}{2\times3} = 16.36\,\Omega \text{ or } 0.978\,\Omega$$

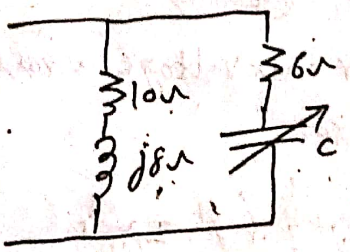
$$\therefore L = \frac{\overset{(X_L)}{16.36}}{\underset{\substack{(\omega)\\(\text{given})}}{5000}} = \underline{\underline{3.27\,mH}} \text{ or } L = \frac{0.978}{5000} = \underline{\underline{0.196\,mH}}$$

$(X_L = \omega L)$

5.20. Find the values of C for which the circuit given in figure resonates at 750 Hz

$x^2 - 3x + 2$

$x^2 + 4x + 5$ (complex eq)



<u>Sol:</u>

$$Y = \frac{1}{10+j8} + \frac{1}{6-jX_c}$$

$$Y = \frac{10-j8}{10^2+8^2} + \frac{6+jX_c}{6^2+X_c^2}$$

$$Y = \left(\frac{10}{164} + \frac{6}{36+X_c^2}\right) + j\left(\frac{X_c}{36+X_c^2} - \frac{8}{164}\right)$$

At resonance, the imaginary part of Y is zero

$$\frac{X_c}{36+X_c^2} - \frac{8}{164} = 0$$

$$164X_c - 8X_c^2 - 288 = 0$$

$$2X_c^2 - 41X_c + 72 = 0 \Rightarrow$$

$$X_c = \frac{41 \pm \sqrt{41^2 - 4\times2\times72}}{2\times2} = \underset{(X_{c1})}{18.56\,\Omega} \text{ or } \underset{(X_{c2})}{1.94\,\Omega}$$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$f = 750\,Hz$ (given) $C_1 = \frac{1}{2\pi f X_{c1}} = \underline{\underline{11.44\,\mu F}}$ $\quad C_2 = \frac{1}{2\pi f X_{c2}} = \underline{\underline{109.45\,\mu F}}$